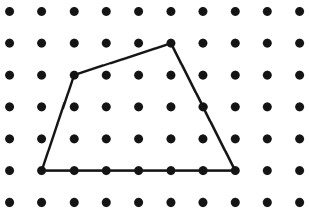
Pick's Formula

You probably know how to find the area of polygonal regions such as squares, rectangles, parallelograms and triangles. But how would you find the area of the dinosaur footprint at the right?

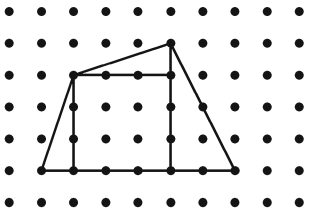
You can place any of the cited polygons on a square dot grid and count squares to find their dimensions and consequently their area. About a hundred years ago, Austrian mathematician Georg Alexander Pick (1859-1943) discovered a relationship, now known as "Pick's Formula", for finding the area of arbitrary polygonal regions on a square dot grid.

Let's begin by looking at two simple polygons on a square dot grid. The dots are called lattice points. For each region, let's count the number of lattice points on the boundary, the number in the interior, and compare our findings to the area that you get by counting squares and using the formulas for the areas of squares, rectangles and triangles.

Polygon A

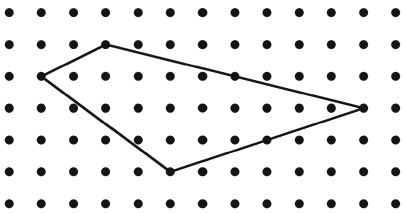


10 boundary points
12 interior points

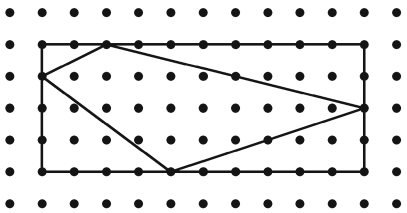


$$A = 3^2 + \tfrac{1}{2}(1)(3) + \tfrac{1}{2}(3)(1) + \tfrac{1}{2}(2)(4)$$
$$= 16$$

Polygon B



6 boundary points
17 interior points



$$A = 10(4) - \tfrac{1}{2}(2)(1) - \tfrac{1}{2}(8)(2) - \tfrac{1}{2}(6)(2) - \tfrac{1}{2}(4)(3)$$
$$= 40 - 1 - 8 - 6 - 6$$
$$= 19$$

Fortunately there is an on-line variant of a common geoboard that will provide more data. When you draw a polygonal region on its square dot grid, the geoboard applet will find the number of boundary points, the number of interior points, and the area.

Exercise

Access the geoboard applet and add data to the following table where B is the number of boundary points, I is the number of interior points, and A is the area of the polygonal region.

B	10	6							
I	12	17							
A	16	19							

1. Study the table for patterns. Do you see any relationship between the three numbers in each column? If you do not, do you see any kind of a relationship between B and A ? A *very simple* relationship involving even and odd numbers?

2. With this simple relationship in mind, add a row to the table beneath the B row with the title $B/2$. Calculate the corresponding data entries.

3. Study the relationship between the entries in the last three rows of each column, that is, between $B/2$, I , and A . Write a formula connecting the three entries.

4. The result is known as "Pick's formula". Use the formula to approximate the area of the dinosaur footprint at the right. Incidentally, the scale is 4 sq = 6 in. If you wish the area of the footprint in square inches, you will have additional calculations to make.