The Klein Bottle is a topological surface discovered by Mr. Klein in 1882. It’s a twisted torus (donut) in the same way that a Möbius Strip is a twisted loop. Even though a Klein Bottle only works properly in four dimensions, it does cast a lovely shadow in three dimensions that can be knitted to make a mighty fine hat.

Klein Bottles have been knitted before (see fantastic examples by Marie-Christine Mahe on knitty.com) but I wanted to raise the mathematical stakes. So I worked with my poor Mum to make a stripy Klein Bottle hat, where each stripe represents a different digit of Pi. Please email me any photos of you wearing this hat.

If there exists a more nerdy hat anywhere, please do let me know.

- Matt Parker

**Matt’s Mum Says:**

This Klein Bottle Pi Hat is knitted in rows using two main contrasting colours to represent each number in Pi, eg: 3.1415... The number 3 will be three rows knitted in one colour (light blue), then use a contrasting wool (pink) to represent the decimal point which is one row (this colour won’t be used again), then the number 1 will be one row in the other main colour (dark blue). Then the next number 4 is represented by four rows using the first main colour again and so on. Zero (0) is represented by one row using a different contrasting colour (light grey). This colour is also used for three rows to represent the “…” after the last number to be knitted (to represent the ongoing numbers).

You could start the Pi at the beginning of the hat but my son wanted it to start just under the folded edge on the inside so that is not seen.

**Materials:**

- DK Wool 1 x 50 gram ball first main colour and 1 x 50 gram ball second main colour
- Two more contrasting colours for the decimal point row and the zero rows
- 1 set DPN’s 4.5 mm
- 2 safety pins
- 4 stitch markers
- tapestry needle and waste yarn

Gauge: 11 sts and 15 rows = 4 inches in 1x1 rib (a little bit either way in the gauge won’t make a huge difference)
Tube of Klein Hat:

Cast on 24 sts (using second main colour)
Divide sts evenly onto 4 DP's and join to begin working in the round, being careful not to twist. Place safety pin in work to mark beginning of round, moving the pin up every few rounds.
Work in 1x1 rib (ie k1, P1 to the end)
Repeat this row until the tube measures about 10 inches (or desired length) remembering to apply the Pi numbers. In the hat I made, I started with the second main colour using it for seven rows then changing to the first main colour to knit the next two rows and so on (to fit in with the way my son wanted the Pi to start just at the bottom of the inside). The tube took the first 15 numbers of the chart of the Pi numbers. This tube forms the loop which curves out of the top of the hat.

Shape Elongated Outer Cap:

Round 1: (K1,P1,K1,M1, place marker, P1,K1,P1,M1) 4 times. 32Sts.
8 sections have been formed, 2 on each needle. Each marker indicates the end of one section; the end of each needle the end of another section
Round 2: work row in rib as set, working new stitches in pattern. I found that I worked the new stitches in what would have come next in the pattern eg K1,P1,K1,(P1 new stitch)marker P1,K1,P1,(K1) end of section/needle.
Round 3: (Work in Rib as set to end of section, M1) 8 times. (ie 8 increases) 40sts.
Repeat Rounds 2 & 3 four more times. 72Sts
Remove stitch markers. Retain safety pin which indicates end of round. Place a second safety pin in the last round worked. Do not move this pin up as you work, as this is for measuring purposes. This round will be referred to as “marked round”.
Work in 1x1 Rib as set until work measures 1 inch from the marked round.

Side Opening:

A small vertical opening is formed in the side of the hat by working several rows back and forth.
Next Row: Turn work and work 1 row in pattern as set with WS of work facing you. You may wish to turn work inside out to do this.
Next Row: Turn work again so that RS is facing and work 1 row in set pattern.
Repeat these 2 rows twice more.
Resume working in the round.

Hat Body:

Work in 1x1 Rib as set following the chart for the rows and colour changes. About 16 inches from the marked round.
Shape Rounded Inner Cap:

Work 1 round in rib as set, placing a marker after first 9 sts on each needle, 8 sections have been formed as for the outer shaping of cap. (This row, is the last row of the 2nd last number 5 in the chart)
Next Round: (SSK, work in rib to end of section) 8 times. 8 sts decreased. 64sts. Repeat this round 5 times.
Work 3 rounds in rib as set, forming a very short tube.
Place remaining sts on waste yarn. Break yarn leaving a 12 inch tail.

Finishing:

Push short tube inside out and to inside of work, folding work inside itself until short tube emerges from side opening.
Use yarn tail to sew held sts to CO end of tube. Be sure each st is secured. (If you chose to use a provisional CO method, graft beginning and end of work together.) Weave in the ends.

Pi is not only an irrational number (because it cannot be written as a factional) but it is also a transcendental number because it can’t be written as the solution to a nice polynomial! Which makes it officially an awesome number.

The decimal places also never end and they never repeat. So if you choose any string of digits, it will appear somewhere in Pi. For example, my birthday appears at the 30,560,847th decimal place of Pi.

In the Klein Hat, the stripes represent the following decimal expansion of Pi:

\[ \pi = 3.141592653589793238462643383279502884197… \]

The “…” at the end represents the fact that Pi will never end. This is so important that I made sure it is represented in the hat as three grey rows. If you wear the hat carefully, the pink row for the decimal point will sit safely away just inside the hat, within easy reach should you need a decimal point in an emergency.